

Assessment Exemplars: Permutations and Combinations

Use the following information to answer the next question

Josh wants to rent a car. He has narrowed his choices to a sedan, a compact or an economy car. The colors available are black, red, white, blue or silver. He may also choose between a standard or an automatic transmission.

1. Determine the total number of options Josh potentially has in his choice of rental vehicle.
2. How many arrangements of all of the letters of the word **REASON** are there if the arrangement must start with an **S**?
3. **SE:** If all of the letters in the word **DIPLOMA** are used, then how many different arrangements are possible that begin and end with an **I, O, or A**?

Use the following information to answer the next question

A volleyball team made up of 6 players stands in a line facing the camera.

4. If Megan and Brooke **must be together**, then how many different arrangements are possible for the picture?
5. If Brianna and Kristy **must be apart**, then how many different arrangements are possible for the picture?
6. **SE:** Determine the number of different arrangements using all the letters of the word **ACCESSES** that
 - begin with exactly two **S**'s.
 - begin with at least two **S**'s.

Use the following information to answer the next question

At a car dealership, the manager wants to line up 10 cars of the same model in the parking lot. There are 3 red cars, 2 blue cars and 5 green cars.

7. **SE:** If all 10 cars are lined up in a row facing forward, determine the number of possible car arrangements if the blue cars **cannot** be together

Use the following information to answer the next question

If 14 different types of fruit are available, how many different fruit salads could be made using exactly 5 types of fruit?

- Student 1** Kevin used $14!$ to solve the problem.
- Student 2** Ron suggested using ${}_{14}P_5$.
- Student 3** Michelle solved the problem using ${}_{14}C_9$.
- Student 4** Jackie thought that ${}_5P_{14}$ would give the correct answer.
- Student 5** Stan decided to use $\binom{14}{5}$.

8. The correct solution would be obtained by student number _____ and student number _____.

9. At a meeting, every person in attendance shakes every other person's hand exactly once. If there are 36 handshakes in total, how many people were at the meeting?

- ____ 10. Using the letters in the word, "**SHAKESPEARE**" how many 11-letter words can be formed if every word must begin and end with an **S**

- a) 15 120
b) 30 240

- c) 1 663 200
d) 3 362 400

11. **SE:** How many different 4-letter arrangements are possible using any 2 letters from the word **SMILE** and any 2 letters from the word **FROG**?

12. Find the value of a if the expansion of $(2x + 3)^{2a-5}$ has 18 terms.

Use the following information to answer the next question

A student made the following statements regarding the expansion of $(a + b)^4$.

- Statement 1** The total number of terms is 5.
Statement 2 The middle term is $6a^2b^2$.
Statement 3 The sum of the leading coefficients of all the terms is 14.
Statement 4 For the term $4a^3b^m$, the value of m is 1.
Statement 5 The leading coefficient of the first term is ${}_4C_1$.

13. The three statements that are true are numbered _____, _____, and _____.
14. Determine the simplified form of the sixth term in the expansion of $(a - 2b)^7$
15. **SE:** In the expansion of $(3a + b^2)^{10}$, what is the coefficient of the term containing a^4b^{12} ?

Use the following information to answer the next question

In the expansion of the binomial $(2a + \frac{1}{a})^8$, the constant term can be found using the general

term $t_{k+1} = {}_nC_k x^{n-k} y^k$. The list below shows possible values for n, k, x , and y .

1	0	4	5	7	a
2	3	5	8	8	a^{-1}
3	4	6	$2a$	9	$-a$

16. **SE:** To find the constant term, the correct values of n, k, x , and y that must be used are numbered, respectively _____, _____, _____, and _____.

Name: _____

ID: A

17. **SE:** Given that a term in the expansion of $(ax - y)^6$ is $-252xy^5$, determine the numerical value of a .
18. Suppose you find out that on the first 12 multiple choice questions on the Diploma Exam Key, there are 3 questions that have the answer **A**, 2 that have the answer **B**, 1 that have the answer **C** and 6 that have the answer **D**. How many possible answer keys could be created using the letters above?
19. The value of the sixth term in the 19th row of Pascal's triangle is _____.
20. The sum of the following sequence; ${}_8C_2 + {}_8C_3 + {}_8C_4 + \dots + {}_8C_7$ is

Assessment Exemplars: Permutations and Combinations Answer Section

1. 30
2. 120
3. 720
4. 240
5. 480
6.
 - begin with exactly two S's. = 150
 - begin with at least two S's. = 150 + 30 = 180
$$\frac{8! \times {}_9P_2}{3!2!5!} = 2\,016$$
7. $\frac{8! \times {}_9P_2}{3!2!5!} = 2\,016$
8. 35 or 53
9. ${}_n C_2 = 36, n = 9$
10. B

$$({}_5 C_2 \times {}_4 C_2) \times 4! = 1\,440$$
11. Choose 2 letters from each word first, and then arrange the 4 letters.
12. 11
13. 124
14. $t_6 = -672a^2b^5$
15.

Since the term must include a^4b^{12} , using the general term, $k = 6$,

$$t_{6+1} = {}_{10}C_6(3a)^{10-6}(b^2)^6$$

$$t_7 = 210(81a^4)(b^{12})$$

$$t_7 = 17\,010a^4b^{12}$$

Therefore, the coefficient of the term is 17 010.
16. 5368

n is the exponent on the binomial; therefore, $n = 8$.

Since the exponent on the variable for the constant term must be zero (a^0), using the general term, $k = 4$.

x is the first term in the binomial; therefore, $x = 2a$.

y is the second term in the binomial; therefore, $y = \frac{1}{a} = a^{-1}$.
17. $a = 42$
18. 55440
19. 8568 (${}_{18}C_5$)
20. 246