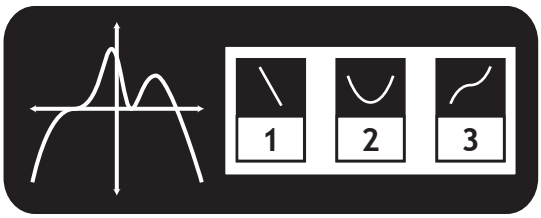


Math 30-1

Polynomial, Radical, and Rational Functions

LESSON ONE - *Polynomial Functions*

Lesson Notes

Example 1 Introduction to Polynomial Functions.

Defining Polynomials

a) Given the general form of a polynomial function:

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x^1 + a_0$$

the leading coefficient is _____.

the degree of the polynomial is _____.

the constant term of the polynomial is _____.

For each polynomial function given below, state the leading coefficient, degree, and constant term.

i) $f(x) = 3x - 2$

leading coefficient: _____ degree: _____ constant term: _____

ii) $y = x^3 + 2x^2 - x - 1$

leading coefficient: _____ degree: _____ constant term: _____

iii) $P(x) = 5$

leading coefficient: _____ degree: _____ constant term: _____

b) Determine which expressions are polynomials. Explain your reasoning.

i) $x^5 + 3$

polynomial: yes no

ii) $5^x + 3$

polynomial: yes no

iii) 3

polynomial: yes no

iv) $4x^2 - 5\sqrt{x} - 1$

polynomial: yes no

v) $x^2 + \frac{1}{3}x - 4$

polynomial: yes no

vi) $|x|$

polynomial: yes no

vii) $5\sqrt{x} - 1$

polynomial: yes no

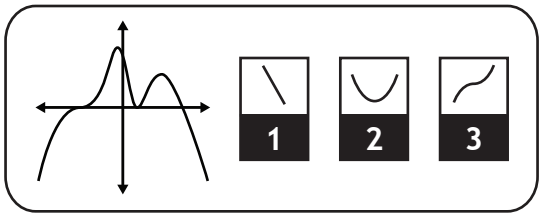
viii) $\sqrt{7}x + 2$

polynomial: yes no

ix) $\frac{1}{x+3}$

polynomial: yes no

Polynomial, Radical, and Rational Functions
LESSON ONE - *Polynomial Functions*
Lesson Notes



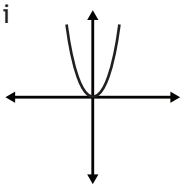
Example 2

End Behaviour of Polynomial Functions.

Even-Degree Polynomials

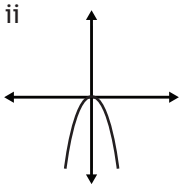
a) The equations and graphs of several even-degree polynomials are shown below. Study these graphs and generalize the end behaviour of even-degree polynomials.

i



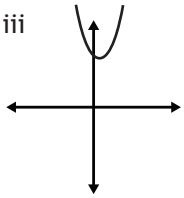
$f(x) = x^2$
quadratic

ii



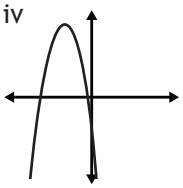
$f(x) = -x^2$
quadratic

iii



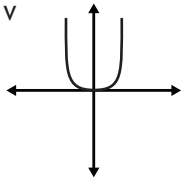
$f(x) = x^2 - x + 6$
quadratic

iv



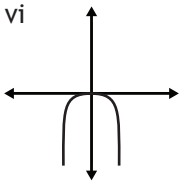
$f(x) = -x^2 - 8x - 7$
quadratic

v



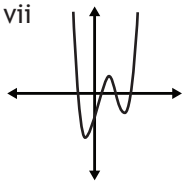
$f(x) = x^4$
quartic

vi



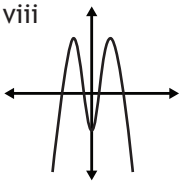
$f(x) = -x^4$
quartic

vii



$f(x) = x^4 - 4x^3 + x^2 + 7x - 3$
quartic

viii



$f(x) = -x^4 + 7x^2 - 5$
quartic

State the End Behaviour of even-degree polynomials:

Sign of Leading Coefficient	*End Behaviour*
Positive	
Negative	

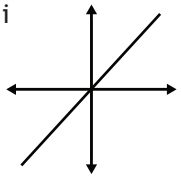
LESSON ONE - *Polynomial Functions*

Lesson Notes

b) The equations and graphs of several odd-degree polynomials are shown below. Study these graphs and generalize the end behaviour of odd-degree polynomials.

Odd-Degree Polynomials

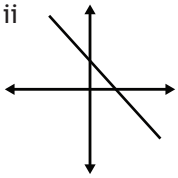
i



$f(x) = x$

linear

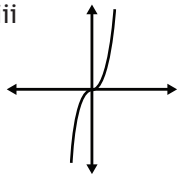
ii



$f(x) = -x + 4$

linear

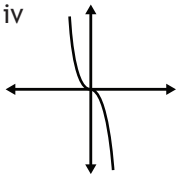
iii



$f(x) = x^3$

cubic

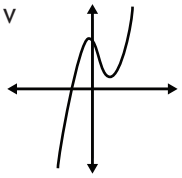
iv



$f(x) = -x^3$

cubic

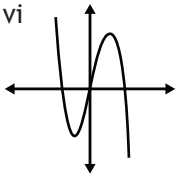
v



$f(x) = x^3 - 2x^2 - 2x + 6$

cubic

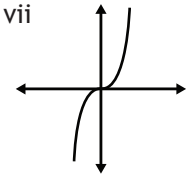
vi



$f(x) = -x^3 + 7x$

cubic

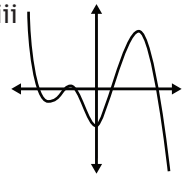
vii



$f(x) = x^5$

quintic

viii



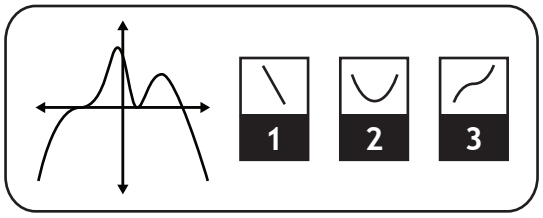
$f(x) = -x^5 - 4x^4 + 40x^3 + 160x^2 - 144x - 576$

quintic

State the End Behaviour of odd-degree polynomials:

Sign of Leading Coefficient	*End Behaviour*
Positive	
Negative	

Polynomial, Radical, and Rational Functions
LESSON ONE - *Polynomial Functions*
Lesson Notes



Example 3

Zeros, Roots, and x-intercepts of a Polynomial Function.

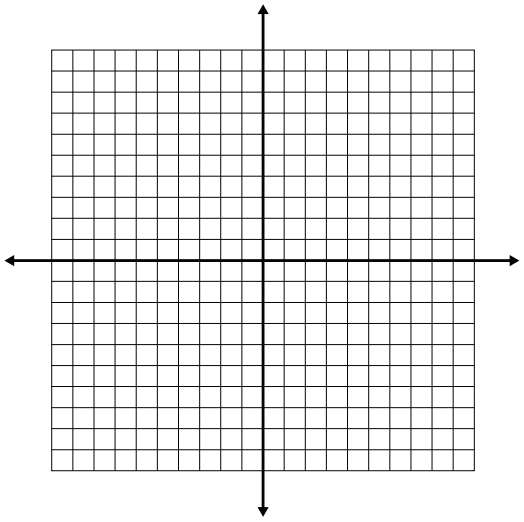
Zeros, roots, and x-intercepts

a) Define "*zero of a polynomial function*". Determine if each value is a zero of $P(x) = x^2 - 4x - 5$.

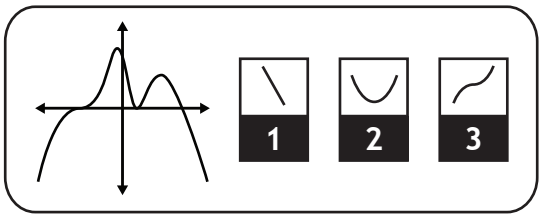
i) -1 ii) 3

b) Find the zeros of $P(x) = x^2 - 4x - 5$ by solving for the roots of the related equation, $P(x) = 0$.

c) Use a graphing calculator to graph $P(x) = x^2 - 4x - 5$. How are the zeros of the polynomial related to the x-intercepts of the graph?



d) How do you know when to describe solutions as zeros, roots, or x-intercepts?



LESSON ONE - *Polynomial Functions*

Lesson Notes

Example 11

Use a graphing calculator to graph each polynomial function. Find window settings that clearly show the important features of each graph *(x-intercepts, y-intercept, and end behaviour)*.

Graphing Polynomials with Technology

a) $P(x) = x^2 - 2x - 168$

Draw the graph.

b) $P(x) = x^3 + 7x^2 - 44x$

Draw the graph.

c) $P(x) = x^3 - 16x^2 - 144x + 1152$

Draw the graph.